GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 6250 Spring 2017
Problem Set #8

Assigned: 27-Mar-17

Due Date: 13-Apr-17

Quiz #3 will be held in class on Wednesday, April 12.

This assignment before midnight on the due date–slip it under Prof. Anderson's door if he is not available.

As stated in the syllabus, unauthorized use of previous semester course materials is strictly prohibited in this course.

PROBLEM 8.1:
Using you class notes, prepare a 1-2 paragraph summary of what we talked about in class in the last week. I do not want just a bulleted list of topics, I want you to use complete sentences and establish context (Why is what we have learned relevant? How does it connect with other things you have learned here or in other classes?). The more insight you give, the better.

PROBLEM 8.2:
Suppose that we want to create a realization of Gaussian noise $\boldsymbol{e} \in \mathbb{R}^5$ with covariance matrix

$$\boldsymbol{R} = \begin{bmatrix} 1 & 1/2 & 1/4 & 1/8 & 1/16 \\ 1/2 & 1 & 1/2 & 1/4 & 1/8 \\ 1/4 & 1/2 & 1 & 1/2 & 1/4 \\ 1/8 & 1/4 & 1/2 & 1 & 1/2 \\ 1/16 & 1/8 & 1/4 & 1/2 & 1 \end{bmatrix}.$$

We have at our disposal a random number generator that creates independent and identically distributed Gaussian random variables with variance 1. We use this to generate $\boldsymbol{e}_{\text{ind}}$, and then pass the output through a matrix to give it the desired covariance structure. Find a matrix $\boldsymbol{Q}$ such that the covariance matrix of $\boldsymbol{Q}\boldsymbol{e}_{\text{ind}}$ is $\boldsymbol{R}$.

PROBLEM 8.3:
Download the file `blocksdeconv.mat`. This file contains the vectors:

- `x`: the 512×1 "blocks" signal
- `h`: a 30×1 boxcar filter
- `y`: a 541×1 vector of observations of h convolved with x
- `yn`: a noisy observation of y. The noise is iid Gaussian with standard deviation .01.

1. Write a function which takes an vector $\boldsymbol{h}$ of length L and a number N, and returns the $M \times N$ (with $M = N + L - 1$) matrix $\boldsymbol{A}$ such that for any $\boldsymbol{x} \in \mathbb{R}^N$, $\boldsymbol{Ax}$ is the vector of non-zero values of $\boldsymbol{h}$ convolved with $\boldsymbol{x}$.

2. Use MATLAB's `svd()` command to calculate the SVD of $\boldsymbol{A}$. What is the largest singular value? What is the smallest singular value? Calculate $\boldsymbol{A}^\dagger \boldsymbol{y}$ and plot it ($\boldsymbol{y}$ is the noise-free data).

3. Apply $\boldsymbol{A}^\dagger$ to the noisy `yn`. Plot the result. Calculate the mean-sqaure error $\|\boldsymbol{x} - \hat{\boldsymbol{x}}\|_2^2$ and compare to the measurement error $\|\boldsymbol{y} - \boldsymbol{yn}\|_2^2$.

4. Form an approximation to $\boldsymbol{A}$ by truncating the last q terms in the singular value decomposition:
$$\boldsymbol{A}' = \sum_{k=1}^{p-q} \sigma_k \boldsymbol{u}_k \boldsymbol{v}_k^{\mathrm{T}}.$$
Apply the new pseudo-inverse $\boldsymbol{A}'^\dagger$ to `yn` and plot the result. Try a number of different values of q, and choose the one which "looks best" to turn in (indicate the value of q used). Calculate the mean-square reconstruction error.

5. Now form another approximate inverse using Tikhonov regularization. Try a number of different values for δ and choose the one which "looks best" to turn in (indicate the value of δ used). Calculate the mean-square reconstruction error.

6. Summarize your findings by comparing the MSE in parts (c), (d), and (e). Also include the error of doing nothing: $\|\boldsymbol{x} - \boldsymbol{yn}'\|_2^2$ where $\boldsymbol{yn}'$ is the appropriate piece of $\boldsymbol{yn}$.

PROBLEM 8.4:

Let

$$\boldsymbol{A} = \begin{bmatrix} 4 & 0 & 1 \\ 5 & 6 & -1 \\ 2 & 4 & -1 \\ 1 & -2 & 1 \\ 8 & -4 & 2 \end{bmatrix}.$$

Suppose that we observe

$$\boldsymbol{y} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{e}$$

where

$$\boldsymbol{y} = \begin{bmatrix} 6.1709 \\ -1.6492 \\ 6.6345 \\ 13.8419 \\ 4.9064 \end{bmatrix},$$

and $\boldsymbol{e}$ has covariance matrix $\boldsymbol{R}$ from question 8.2.

1. Find the best linear unbiased estimate $\hat{\boldsymbol{x}}_{\text{blue}}$ of $\boldsymbol{x}$.
2. What is the mean squared error of your estimate $\mathrm{E}[\|\boldsymbol{x} - \hat{\boldsymbol{x}}_{\text{blue}}\|_2^2]$?