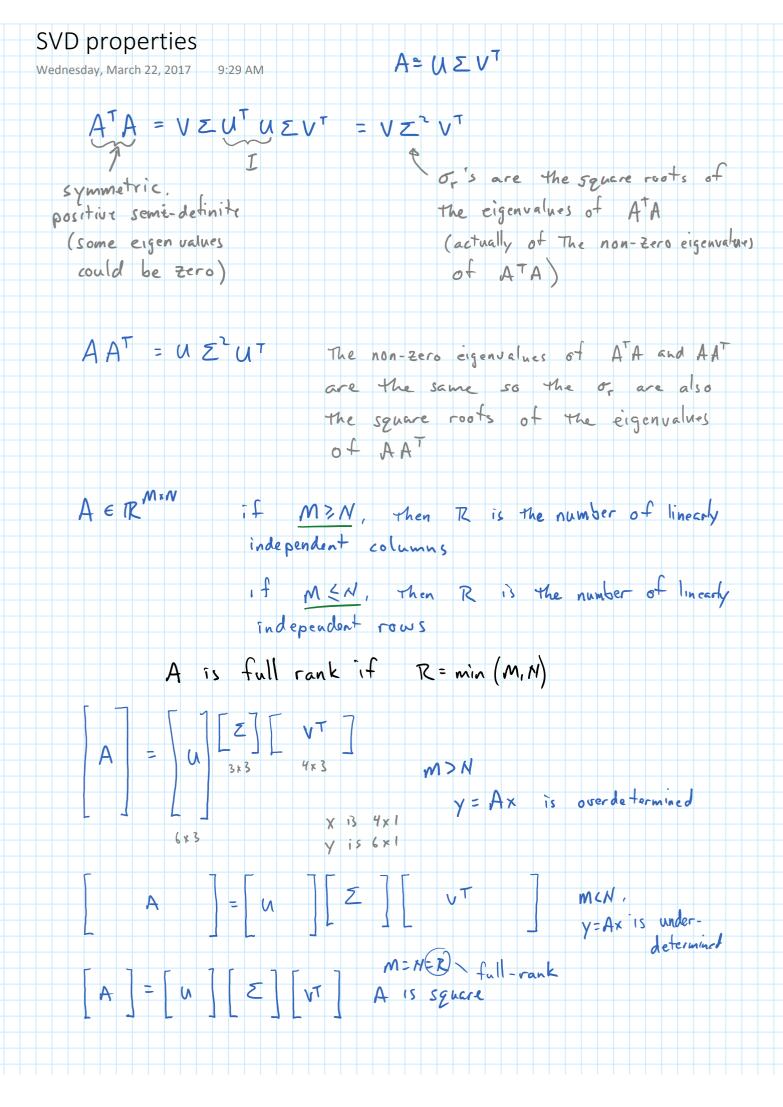


Singular Value Decomposition

Wednesday, March 22, 2017 9:15 AM

We generally are interested in non-sym+det matrices. $y = A \times , \quad y \in \mathbb{R}^{M}, \quad A \text{ is } M \times N, \quad x \in \mathbb{R}^{N}$ The singular value decomposition (SVD) of A is A= UZVT $U = [U_1 | U_2 | \cdots | U_R]$ is an MxR metrix um EIR are orthogonal and normalized so that UU= J. Note UUT = J when RCM in general. EUm3 form an orthobasis for the range space of A $V = [v_1 | v_2 | \dots | v_R]$ is an NXR matrix v, ERN and are orthonormal $V^{T}V = I$ but, in general, $VV^{T} \neq I$ when R < NEVn 3 form an orthobasis for the range space of AT -> Range (AT) consists of everything which is orthogonal to the null space of A. E is an RXR diagonal matrix with positive entries $\mathcal{Z} = \begin{bmatrix} \sigma_1 & & \\ & \sigma_2 & \\ & & \\ & & & & \\ & & & \\ &$



SVD and Least Squares

Wednesday, March 22, 2017 9:44 AM

"Solving" y = Ax using SUD $y \in \mathbb{R}^{M}$ $A \in \mathbb{R}^{M \times N}$ $x \in \mathbb{R}^{N}$

 $A = \sum_{r=1}^{R} \sigma_r u_r V_r^{T}$

Given y, we want to find x in such a way that 1. when the unique solution exists, use that

z. when no solution exists, return something reasonable

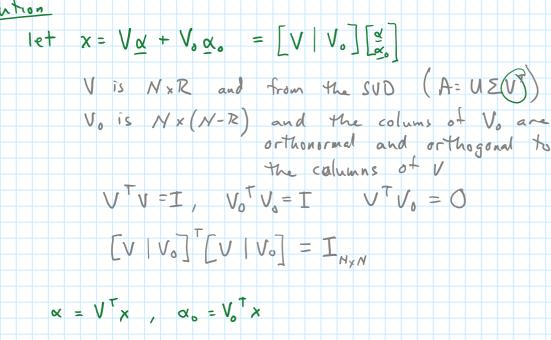
Since $A = U \geq V^T = \begin{bmatrix} u_1 & u_2 & \cdots & u_R \end{bmatrix} \begin{bmatrix} \sigma_1 & \sigma_2 & \cdots & \sigma_r \\ & & & & & \\ & & & & & \\ & & &$

3. When there are an infinite number of solutions, choose the "best" one.

Define a residual Least-Squares framewirk

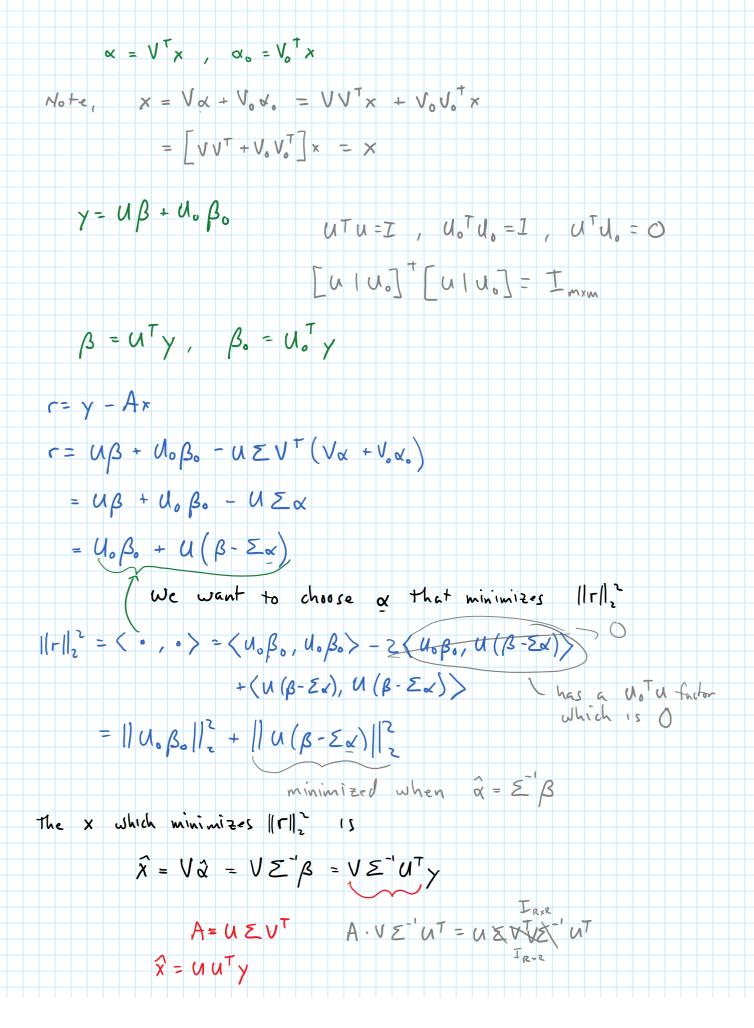
 $r = y - Ax \Rightarrow optimize min || y - Ax ||_2 x \in \mathbb{R}^N$

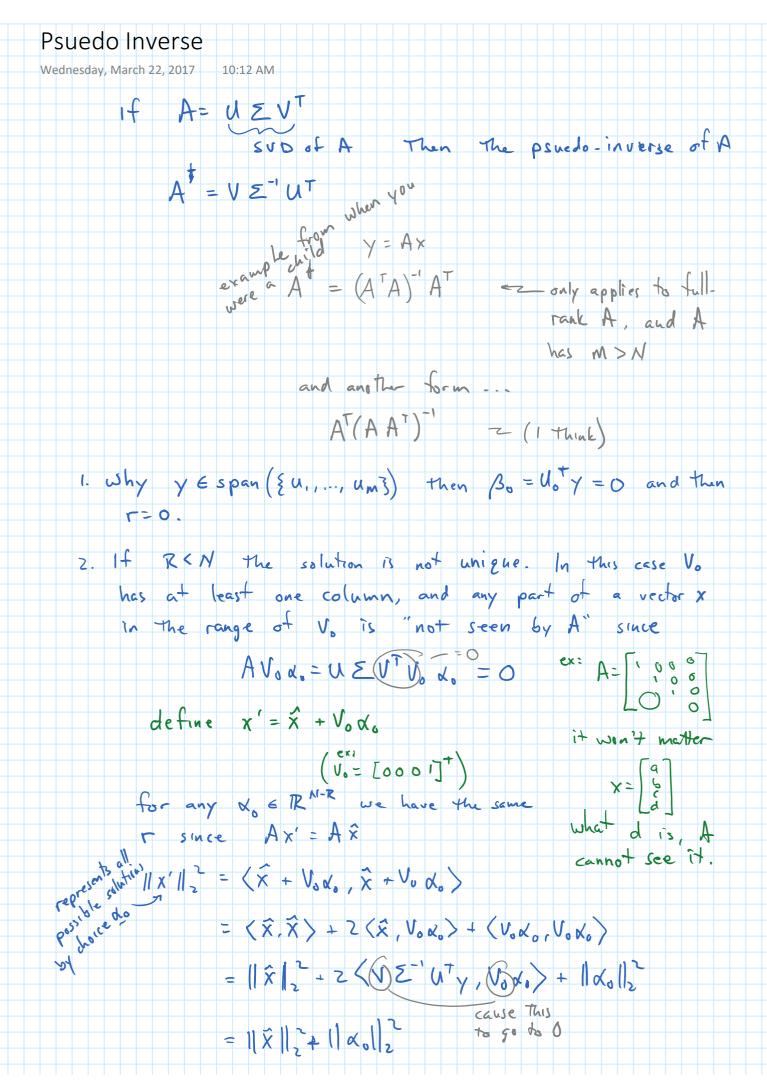
Solution



SVD and Least Squares cont.

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SVD Least-Squares Solution

Wednesday, March 22, 2017 10:27 AM

Using the SVD to find the solution as described on the previous pages minimizes both the residual **and** the norm of the solution!

in Matlab $x = A \setminus y;$